The difference of differential (5) times $N_p'(x)$ and differential (6) times $J_p'(x)$ gives

$$J_p'''(x)N_p'(x) - N_p'''(x)J_p'(x) = -\frac{6p^2}{\pi x^4} + \frac{2}{\pi x^2}. \quad (8)$$

Similarly

$$J_p^{IV}(x)N_p'(x) - N_p^{IV}(x)J_p'(x) = \frac{2p^4}{\pi x^5} + \frac{22p^2}{\pi x^5} - \frac{4p^2}{\pi x^3} - \frac{6}{\pi x^3} + \frac{2}{\pi x} \quad (9)$$

$$J_p^{V}(x)N_p'(x) - N_p^{V}(x)J_p'(x) = -\frac{20p^4}{\pi x^6} - \frac{100p^2}{\pi x^6} + \frac{24p^2}{\pi x^4} + \frac{24}{\pi x^4} - \frac{4}{\pi x^2} \cdots \text{etc.} \quad (10)$$

If $(x_i - x)$ is very small, the higher power terms of $(x_i - x)$ may be neglected for the approximate computation. Substituting (7)–(9) into (4) and dropping the terms higher than $(x_i - x)^2$ yields

$$(x_i - x)^2p^4 + (11x_i^2 - 31x_ix + 26x^2 - 2x_i^2x^2 + 4x_ix^3 - 2x^3)p^2 + (x^6 - 2x_ix^5 + x_i^2x^4 - 3x_i^2x^2 + 9x_ix^3 - 12x^4) = 0. \quad (11)$$

Substituting (7)–(10) into (4) and dropping the terms higher than $(x_i - x)^3$ gives

$$p^4\left[(x_i - x)^2 - \frac{5(x_i - x)^3}{2x}\right] + p^2\left\{6x^2 - 9x(x_i - x) + (11 - 2x^2)(x_i - x)^2 + (x_i - x)^3\left[-\frac{25}{2x} + 3x\right]\right\} + \left\{3x^3(x_i - x) + (x^4 - 3x^2)(x_i - x)^2 - 6x^4 + (x_i - x)^3\left[3x - \frac{x^3}{2}\right]\right\} = 0. \quad (12)$$

Sample of calculation: if $x_i = 0.6566$ and $x = 0.544375$, substituting x_i and x into (11) yields

$$p = \pm 0.59495 \quad \text{and} \quad p = \pm j10.406;$$

substituting x_i and x into (12) yields

$$p = \pm\, 0.60078 \quad \text{and} \quad p = \pm j14.769.$$

The real roots computed from the Bessel function series expansion are ± 0.599 and are close to the values obtained from (11) and (12). The first pairs of complex roots obtained with (11) are different from those obtained with (12). The complex roots of (4) must be solved with more higher-power terms to obtain a close value. However, these complex roots are large and the field will attenuate quickly.

Acknowledgment

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A Source of Error in the Use of Slope Detection for Perturbation Measurements*

In a typical laboratory measurement of the field-intensity distribution in a resonant cavity, the resonant frequency of the cavity is determined as a function of the position of a small metal or dielectric bead. The field distribution itself is then deduced directly from the frequency change.

A simple scheme advanced by Ayers, Chu and Gallagher[1] depends on the fact that a graph of the logarithm of the response of a cavity vs frequency is very nearly linear between 1 and 7 db below the maximum response. In this region, the frequency perturbation caused by a bead is directly proportional to the change of attenuation through the cavity with a maximum error of ± 1 per cent of the range. A fixed-frequency source is tuned slightly off the unperturbed resonance of the cavity; any of a number of circuits for measuring the change of insertion loss of the cavity may be used; and a large amount of data may be taken quickly.

This scheme of "slope detection" has one disadvantage: the power range in which the variation is linear is only 6 db, and high precision is difficult to attain. It would appear reasonable to relinquish the linear property and to use larger perturbations which reduce the signal transmissions through the cavity as much as 20 db. It would be required, of course, that the Q and coupling to the cavity remain unchanged.

It is the purpose of this note to demonstrate that the effective coupling is changed, however, if the frequency perturbation is much larger than the unperturbed bandwidth of the cavity. The phase shift introduced by the bead can cause the excitation at one end of the cavity to be greater than that at the other so that, for a given excitation at the vicinity of the input coupling loop, the total stored energy in the cavity may vary. Thus, the coupling efficiency of the loop and the transmission efficiency of the cavity no longer follow the first-order predictions.

Consider the cavity of Fig. 1, consisting of a section of uniform transmssion line of length L symmetrically coupled by ideal transformers of turns-ratio n arranged to produce a voltage maximum at each end of the cavity. The propagation constant at resonance is $\gamma = \alpha + j\beta$. A small change of the operating frequency will produce little change of the attenuation constant α but will change the phase constant to a new value $\beta + \epsilon$, where ϵ is proportional to the frequency change. A small perturbing bead with normalized admittance jB is introduced at a distance l from one end of the cavity. The effect of the bead may be described by computing the over-all transmission coefficient S_{12} of the circuit.[2]

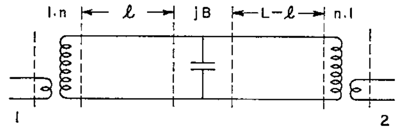

Fig. 1—Equivalent circuit of cavity.

For the usual case of a weakly coupled cavity, one finds, to first-order in $\Delta = \alpha l$,

$$|S_{12}|^2 = \frac{k(1 - \Delta/2)}{1 + (\delta + b\cos^2\beta l)^2 + \frac{1}{2}\Delta b^2\cos^3\beta l[4(1 - 2l/L)\sin\beta l - \cos\beta l]}, \quad (1)$$

where $k = 1/n^2\alpha L$ is the coupling coefficient, $b = B/\alpha L$ is a renormalization of the bead susceptance, Δ is a small quantity which represents the loss within the cavity, and δ is the frequency shift, normalized to the half-bandwidth of the cavity. Note that $\cos \alpha l$ represents the unperturbed electric field distribution.

It is the last term in the denominator which leads to the distortion of the response. It appears to be small since it is of order, Δ, but the term is also proportional to b^2. From the first term, it will be recognized that b is the ratio of the maximum frequency perturbation introduced by the bead to the half-bandwidth of the cavity. If the bead is large enough to shift the resonant frequency several bandwidths, the factor b^2 may well be large enough to make the factor in $1 - 2l/L$ significant. (It should be noted that, since the frequency variable δ does not enter this term, the resonant frequency itself continues to follow the first-order perturbation theory.)

This effect has been observed in tests of $2\pi/3$ mode disk-loaded accelerator sections. For a 3-disk cavity, one wavelength long, a frequency perturbation curve of the form of Fig. 2(a) was observed. The data were obtained using slope detection and a large bead which introduced a maximum change of transmission of -19 db. The curve appeared to agree with expectations except for the end effect where the bead interacts with its image in the end-plate. To eliminate the end-plate errors, a second measurement was made with a 6-disk (two-wavelength) cavity. It was expected that the curve would be identical to two of the original curves pasted together except, of course, for the elimination of end-plate effects on the central hump. Instead, the curve was found to be considerably distorted, as shown in Fig. 2(b).

In the latter cavity the voltage attenuation was relatively large, so that $\Delta = \alpha L$

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[1] W. R. Ayers, E. L. Chu, and W. J. Gallagher, "Measurements of Interaction Impedance in Periodic Circuits," W. W. Hansen Labs. of Physics, Stanford University, Stanford, Calif., Internal Memo., ML Rept. No. 403; June, 1957.

[2] This analysis is presented without regard for space-harmonics in the cavity. The author has shown in a recent paper, "A Perturbation Technique for Impedance Measurements" (presented at the IEE Conf. on Microwave Measurements and Techniques, London, Eng., September 6, 1961), that the presence of space harmonics modifies the effective susceptance of the bead in a manner strictly periodic with the loading elements of the structure.

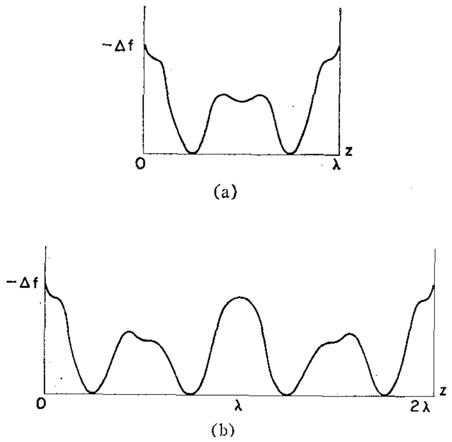

Fig. 2—Experimental data. (a) 3-disk cavity. (b) 6-disk cavity.

$\approx 1/20$. The cavity bandwidth was 343 kc at 2856 Mc; the Q was about 8300. The experimental data in Fig. 2 were taken with a large bead which detuned the cavity resonance 1.62 Mc, about nine times the half-bandwidth. Thus $b=9$, $b^2=81$, and the last term in the denominator of (1) is found to add as much as 25 per cent to the signal on one side of a maximum and subtract the same amount on the other side. The correction does indeed account for the observed results.

In conclusion, it has been shown that a perturbing bead does not affect the coupling to the cavity so long as the maximum perturbation of the resonant frequency is of the order of the bandwidth of the cavity. With a perturbation amounting to several cavity bandwidths, the effective coupling to the cavity can be drastically changed, and makes the slope-detection scheme a poor method of taking the required data. A method which measures the resonant frequency directly is not affected by this phenomenon.

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A New Microwave Mixer Suitable for Use with Very High Intermediate Frequencies*

In the conventional microwave mixer it is generally necessary to provide a low-pass filter or some similar network in the intermediate-frequency output circuit. This filter usually takes the form of a capacitance or choke, which is arranged to present, as nearly as possible, a short circuit to the signal and local oscillator frequencies. For a microwave mixer with an IF of less than 100 Mc chokes have been found to be satisfactory provided that a correct choice of the short-circuit position has been made. It is difficult, however, to design a choke system with a capacitance of less than 5 pf and, even at intermediate frequencies as high as 100 Mc, a capacitance of this value could seriously limit the fractional IF bandwidth or the noise performance attainable. At intermediate frequencies greater than 1000 Mc it was expected that the problems of design of a low-pass filter would be even greater because of the difficulty of meeting the conflicting requirements of efficient RF rejection and IF coupling.

The mixer described[1] uses a pair of crystal diodes arranged in a biphase circuit, which enables the IF output connection to be made in such a way that it is not coupled to the signal and local oscillator power. The decoupling arises from the symmetrical disposition of the two crystals and the fact that the IF output connection is normal to the electric fields of the signal and local oscillator waves, both of which are propagated in the dominant waveguide mode. This configuration eliminates the necessity for chokes in the output circuit. With this arrangement it is possible for power at even harmonics of the signal and local oscillator frequencies to couple to a coaxial output circuit. This could partly account for the somewhat low conversion efficiency measured, but no other indication of the effects of such coupling was observed.

A Biphase Mixer for the Band 11.5–18 Gc

An experimental mixer for use over this band was constructed in WG 18 (RG91U) waveguide; coaxial crystal diodes (type VX3282) were mounted on the opposite broad faces of the waveguide. The center pins of the two crystals were connected by a post extending across the waveguide. The IF output was taken from the center of this post to a BNC connector mounted in the center of the narrow waveguide wall, the lead thus being normal to the dominant mode electric field within the waveguide. The post and crystals were offset from the center line of the waveguide in order to improve the input VSWR (Fig. 1). An adjustable back piston was placed behind the plane of the crystals and post.

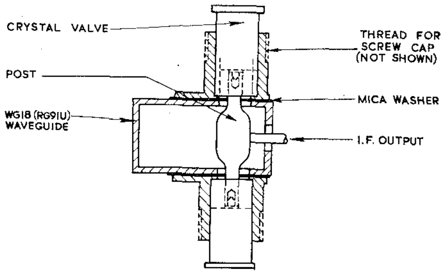

Fig. 1—Arrangement of diodes.

The VSWR of the holder was measured, using a reflectometer, and found to vary between 0.3 and 0.7 over the frequency band. The RF leakage from the hole in the side wall of the mixer, before the BNC connector was placed in position, was found to be negligible. In the first model, no attempt was made to match the IF output circuit, and the VSWR, looking into this port, was found to be about 0.2 when the crystals were operated at 0.15-v bias and 2-ma total current.

Measurements were made of the noise figure of a receiver incorporating this mixer, and using a superheterodyne receiver as the IF amplifier, whose double-sideband noise figure varied from 8.9 db to 14.2 db over the tuning range 1.7 to 2.5 Gc.

The over-all noise figure of the complete receiver, measured over a wide range of RF and IF frequencies, was found to be between 21 and 26 db.

In order to improve the performance an attempt was made to match the IF output circuit, using a shunt stub and a transforming section (Fig. 2); by this means, a match varying between 0.35 and 0.88 over the band was obtained.

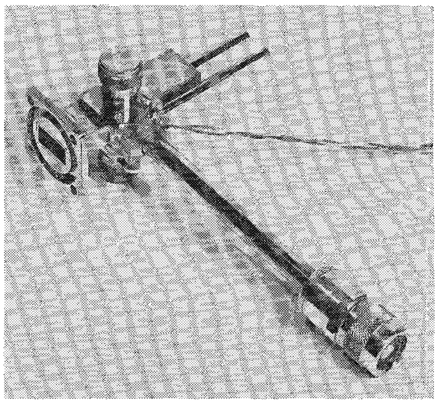

Fig. 2—Experimental mixer.

TABLE I

Local Oscillator Frequency (Gc)	Mean Over-all Noise Figure (db)	Mean Image-Matched Conversion Loss (db)
11.5	17.4	8.5
13.0	17.6	8.5
16.0	15.8	6.5
17.0	15.7	7.0
18.0	17.8	8.5

IF =2.42 Gc.
Noise figure of IF amplifier =9.2 db.

Measurements of receiver noise figure, conversion loss, and effective noise-temperature ratio were made using noise tubes as signal sources at RF and IF frequencies. The results of these measurements are given in Table I.

An estimate of the contribution to the conversion loss arising from the presence of the parasitic barrier capacitance and spreading resistance indicates that the conversion loss at 2 Gc in the crystals alone should be insignificantly different from that at, say, 45 Mc. The discrepancy observed in the present mixer is thought to be due to the effects of mismatch and dissipative loss in the IF circuit. This is confirmed by the fact that the VSWR in the IF circuit was better than expected. Measurements of the effective noise-temperature ratio at the output plug of the biphase mixer gave a mean value of 1.4. This somewhat low value is probably also produced by the loss in the output circuit. For comparison, the measured noise-temperature ratio and image-matched con-

* Received by the PGMTT, November 11, 1961.

[1] British Patent Application No. 25881/60.